Tutor Marked Assignment
THERMAL PHYSICS AND STATISTICAL MECHANICS

Course Code: BPHCT-135
Assignment Code: BPHCT-135/TMA/2026
Max. Marks: 100

Note: Attempt all questions. The marks for each question are indicated against it.

PART A

1. a) Calculate v_{rms} for helium atoms at 300K. At what temperature will oxygen molecules have the same value of v_{rms}? Take $m_{He} = 6.67 \times 10^{-27}\,\text{kg}$. (5)

   b) What is most probable speed? Using the expression of the number of molecules in a Maxwellian gas in the range v to $v + dv$ as

   $$dN_v = 4\pi N\left(\frac{m}{2\pi k_B T}\right)v^2 \exp\left[-\left(\frac{mv^2}{2k_B T}\right)\right]dv$$

   Obtain an expression for most probable speed. (5)

   c) Define mean free path? Obtain an expression of mean free path for zeroth order approximation. (5)

   d) What is Brownian motion? Write its two examples. Show that during sedimentation, particles concentration decreases exponentially as height increases. (5)

   e) Write the expressions of an ideal gas and van der Waals' equation. 10 moles of nitrogen gas occupy a volume of $5 \times 10^{-3}\,\text{m}^3$ in a vessel at $27°\text{C}$. Calculate the pressure exerted by the molecules, if it is assumed to obey van der Waals' equation. Compare this Value with that obtained using ideal gas law.

   Given: $a = 1.39 \times 10^{-6}$ atm $\text{m}^6\,\text{mol}^{-2}$,
   $b = 39.1 \times 10^{-6}\ \text{m}^3\,\text{mol}^{-1}$
   and $R = 8.2 \times 10^{-5}\,\text{atm}\ \text{m}^3\,\text{K}^{-1}\,\text{mol}^{-1}$. (5)

2. a) A mass of an ideal gas $(\gamma = 1.5)$ at $27°\text{C}$ is suddenly compressed to four times its original pressure. Calculate the final temperature of the gas. (5)

   b) i) What are intensive and extensive variables for thermodynamic systems? Write two examples of each. (3)

   ii) Represent an (a) isobaric (b) isochoric (c) isothermal and (d) a cyclic process on p-V indicator diagram. (2)

   c) State the zeroth law of thermodynamics. Explain how the zeroth law introduces the concept of temperature. (5)

   d) Obtain the expressions of work done for an ideal gas (i) in an isothermal expansion (ii)) in an adiabatic process. (5)

   e) Obtain an expression of coefficient of volume expansion for one mole of van der Waals' gas. (5)

PART B

3. a) Consider that two ideal gases having n_1 moles are n_2 moles are enclosed in two separate containers at constant temperate T and pressure P. It these two gases mix, obtain an expression for the entropy of mixing per mole of mixture. (5)

   b) Draw p-V diagram of the Carnot Cycle and hence obtain an expression of efficiency. (5)

   c) Using the Maxwell's relations of thermodynamics, obtain the first and second $T\mathrm{d}S$-equations. (5)

   d) Derive Clausius-Clayperon equation of phase transition. Draw phase diagram of water. (5)

   e) What is Joule-Thomson effect? Write any four findings of Joule- Thomson experiment. (5)

4. a) Derive an expression of the partition function for a single particle monatomic ideal gas. Write the expression for a monatomic gas consisting of N-identical yet distinguishable and non-interacting particles. (5)

   b) N particles obey MB distribution. They are distributed among three states with energies $E_1 = 0$, $E_2 = k_B T$ and $E_3 = 3k_B T$. If the equilibrium energy of the system is $2000\,k_B T$, calculate the total number of particles, N. (5)

   c) Using the Planck's radiation law:

   $$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left(\frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right)-1}\right)d\lambda$$, deduce (i) Rayleigh-Jean's law, (ii) Wien's law and (iii) Stefan's law. (10)

   d) What is Fermi-energy? Obtain an expression for Fermi energy for a system of N Fermions enclosed in a volume V. What will be the value of Fermi-energy (in eV) of gold at Fermi-temperature $6.42\times10^4\ K$? (5)

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1. a) Calculate v_{rms} for helium atoms at 300K. At what temperature will oxygen molecules have the same value of v_{rms} ? Take $m_{He} = 6.67 \times 10^{-27}$ kg. (5)

Answer

For a gas molecule, the root mean square speed is

$$v_{rms} = \sqrt{(3k_B T/m)}.$$

For helium, T = 300 K, $m_{He} = 6.67 \times 10^{-27}$ kg and $k_B = 1.38 \times 10^{-23}$ J K^{-1}. Therefore,

$$v_{rms} = \sqrt{[(3 \times 1.38 \times 10^{-23} \times 300)/(6.67 \times 10^{-27})]} \approx 1.365 \times 10^{3} \text{ m s}^{-1}.$$

Thus the rms speed of helium atoms at 300 K is about 1.37×10^{3} m s^{-1}.

For oxygen molecules to have the same rms speed, T/m must have the same value. An oxygen molecule has molecular mass 32 u, while a helium atom has mass about 4 u. Hence $m_{O2} = 8m_{He}$.

$$T_{O2}/m_{O2} = 300/m_{He} \quad T_{O2} = 8 \times 300 = 2400 \text{ K}.$$

b) What is most probable speed? Using the expression of the number of molecules in a Maxwellian gas in the range v to v + dv as

$$\mathbf{dN_v = 4\pi N (m/2\pi k_B T)^{3/2} v^2 \exp[-mv^2/(2k_B T)] dv}$$

Obtain an expression for most probable speed. (5)

Answer

The most probable speed is the speed possessed by the largest number of molecules in a gas in thermal equilibrium. It is the value of v at which the Maxwell speed-distribution curve has its maximum.

All factors in the given expression other than the speed-dependent part are constant at fixed temperature. Therefore we maximize

$$f(v) = v^2 \exp[-mv^2/(2k_B T)].$$

At the maximum, df/dv = 0. Differentiating,

$$df/dv = \exp(-av^2)[2v - 2a\, v^3] = 0, \quad \text{where} \quad a = m/(2k_B T).$$

Ignoring the trivial solution v = 0, we obtain $1 - a\, v^2 = 0$. Hence

$$v_p^{\,2} = 1/a = 2k_B T/m, \quad \text{so} \quad v_p = \sqrt{(2k_B T/m)}.$$

Thus the most probable speed increases as $\sqrt{T}$ and decreases as $1/\sqrt{m}$.