Course Code	**:**	**BCS-012**
Course Title	**:**	**Basic Mathematics**
Assignment Number	**:**	**BCA_NEW/BCAOL(I)012/Assign/2026-27**
Maximum Marks	**:**	**100**
Last Date of Submission	**:**	**31st October,2026 (For July 2026 Session)**
	:	**30th April, 2027 (For January 2027 Session)**

Note: This assignment has 20 questions of 80 marks (each question carries equal marks). Answer all the questions. Answer all the questions. Rest 20 marks are for viva voce. You may use illustrations and diagrams to enhance explanations. Please go through the guidelines regarding assignments given in the Programme Guide for the format of presentation.

Q1: If A = [3 −1; 2 1], show that $A^2 - 4A + 5I_2 = 0$. Also find A^4.

Q2: Solve the following system of equations using the matrix inverse method:

$3x + 4y + 7z = 14$
$2x - y + 3z = 4$
$2x + 2y - 3z = 0$

Q3: Use the principle of mathematical induction to prove that:
$1/(1\times2) + 1/(2\times3) + \ldots + 1/[n(n+1)] = n/(n+1)$

Q4: If the p^{th} term of an A.P. is q and the q^{th} term of the same A.P. is p, find its r^{th} term.

Q5: How many terms of the G.P. $\sqrt{3}, 3, 3\sqrt{3}, \ldots$add up to 39?

Q6: Find the sum of all integers between 100 and 1000 that are divisible by 7.

Q7: If 1, ω, ω^2 are the cube roots of unity, show that:$(2 - \omega)(2 - \omega^2)(2 - \omega^{19})(2 - \omega^{23}) = 49$

Q8: Use De Moivre's theorem to find the value of $(\sqrt{3} + i)^3$

Q9: If α and β are the roots of $x^2 - 3ax + a^2 = 0$, find the value of a when $\alpha^2 + \beta^2 = 7/4$.

Q10: If α and β are the roots of $2x^2 - 3x - 5 = 0$, form the quadratic equation whose roots are α^2 and β^2.

Q11: Solve the equation: $x^3 - 13x^2 + 15x + 189 = 0$, given that one root exceeds another by 2.

Q12: Solve the inequality and graph its solution: $2/(x - 1) > 5$

Q13: If $y = \log[(\sqrt{(1+x)} - \sqrt{(1-x)})/(\sqrt{(1+x)} + \sqrt{(1-x)})]$, find dy/dx.

Q14: If $y = ae^{mx} + be^{-mx}$, prove that: $d^2y/dx^2 = m^2y$

Q15: Determine the intervals in which $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is increasing and decreasing.

Q16: Find the points of local maximum and local minimum of: $f(x) = x^3 - 6x^2 + 9x + 2014$

Q17: Evaluate: $\int x^2\sqrt{(5x - 3)}\, dx$

Q18: Using integration, find the length of the curve $y = 3 - x$ from $(-1, 4)$ to $(3, 0)$.

Q19: Find the scalar component of projection of the vector $a = 2\hat{i} + 3\hat{j} + 5\hat{k}$ on the vector $b = 2\hat{i} - 2\hat{j} + \hat{k}$.

Q20: A tailor needs at least 40 large buttons and 60 small buttons. A box contains 6 large and 2 small buttons, and a card contains 2 large and 4 small buttons. If the cost of a box is \$3 and the cost of a card is \$2, formulate and solve the problem as a linear programming problem to minimize expenditure.

Q1: If A = [3 −1; 2 1], show that $A^2 - 4A + 5I_2 = 0$. Also find A^4.

Answer

The matrix is A = [[3, −1], [2, 1]], and I_2 is the identity matrix of order 2. We first multiply A by itself.

A^2 = [[3(3) + (−1)(2), 3(−1) + (−1)(1)], [2(3) + 1(2), 2(−1) + 1(1)]]

A^2 = [[7, −4], [8, −1]].

4A = [[12, −4], [8, 4]], $5I_2$ = [[5, 0], [0, 5]].

$A^2 - 4A + 5I_2$ = [[7 − 12 + 5, −4 + 4], [8 − 8, −1 − 4 + 5]] = [[0, 0], [0, 0]].

Thus $A^2 - 4A + 5I_2 = 0$, as required. From this relation, $A^2 = 4A - 5I_2$. Multiplying by A gives

$A^3 = 4A^2 - 5A = 4(4A - 5I_2) - 5A = 11A - 20I_2$.

$A^4 = 11A^2 - 20A = 11(4A - 5I_2) - 20A = 24A - 55I_2$.

A^4 = [[72 − 55, −24], [48, 24 − 55]].

Therefore, A^4 = [[17, −24], [48, −31]].

Q2: Solve the following system of equations using the matrix inverse method:
3x + 4y + 7z = 14
2x − y + 3z = 4
2x + 2y − 3z = 0

Answer

Write the equations in the form AX = B, with the coefficient matrix A and the column vectors X and B:

A = [[3, 4, 7], [2, −1, 3], [2, 2, −3]], X = $[x, y, z]^T$, B = $[14, 4, 0]^T$.

The determinant of A, expanded along its first row, is

|A| = 3[(−1)(−3) − 3(2)] − 4[2(−3) − 3(2)] + 7[2(2) − (−1)(2)]

= 3(−3) − 4(−12) + 7(6) = −9 + 48 + 42 = 81.

Since |A| = 81 ≠ 0, the inverse exists. Evaluating the cofactors and transposing the cofactor matrix gives

adj(A) = [[−3, 26, 19], [12, −23, 5], [6, 2, −11]].

A^{-1} = (1/81) [[−3, 26, 19], [12, −23, 5], [6, 2, −11]].

Now X = A^{-1}B. Multiplying the inverse matrix by B gives